Exercises from Notes on the Exercises

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1. [*00*] What does the rating "*M20*" mean?

According to the author, the rating "*M20*" indicates a mathematically-oriented exercise of medium difficulty requiring perhaps a quarter hour to finish.

2. [*10*] Of what value can the exercises in a textbook be to the reader?

Exercises in a textbook facilitate learning the material by requiring the reader to apply the information read to specific problems. Often, material learned in this way is learned best, as it has been discovered for one's self.

3. [*14*] Prove that $13^3 = 2197$. Generalize your answer. [This is an example of a horrible kind of problem that the author has tried to avoid.]

First, we will prove that $13^3 = 2197$.

Proposition. $13^3 = 2197$.

Proof. We will show that $13^3 = 2197$. By the laws of exponents, the identity function for exponentiation, and simple arithmetic, we have:

$$\begin{aligned} 13^3 &= 13^1 \cdot 13^2 \\ &= 13^1 \cdot 13^1 \cdot 13^1 \\ &= 13 \cdot 13 \cdot 13^1 \\ &= 169 \cdot 13^1 \\ &= 169 \cdot 13 \\ &= 2197 \end{aligned}$$

Therefore, $13^3 = 2197$. $\square$

We may generalize this particular case by noting that $a^3 = a \cdot a \cdot a = \prod_{1 \leq k \leq 3} a$; and in general, that $a^n = \prod_{1 \leq k \leq n} a$ for all positive integers n.

4. [*HM45*] Prove that when n is an integer, $n > 2$, the equation $x^n + y^n = z^n$ has no solution in positive integers x, y, z.

n.a.

See W. J. LeVeque, *Topics in Number Theory* **2** (Reading, Mass.: Addison-Wesley, 1956), Chapter 3; P. Ribenboim, *13 Lectures on Fermat's Last Theorem* (New York: Springer-Verlag, 1979); A. Wiles, *Annals of Mathematics* (2) **141** (1995), 443-551.